On the Stability of Rational, Inbound-Dependent Interdomain Route Selection *

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Abstract

The recent discovery of instability caused by the interaction of local routing policies of multiple ASes has led to extensive research on the subject. However, previous studies analyze stability under a specific route selection algorithm, and focus only on egress route selection. In this paper, we analyze the stability of interdomain routing under two extensions. First, we investigate the general model that the preference of an AS depends on not only its egress routes to the destinations but also its inbound traffic pattern. Second, instead of studying a specific route selection algorithm, we study a general class of route selection algorithms which we call rational route selection algorithms. *We present a sufficient condition to guarantee routing convergence in a heterogeneous network where each AS runs any rational route selection algorithm. We also show that there exist networks which will have persistent route oscillations even when the ASes strictly follow the constraints imposed by business considerations, and adopt* any *rational route selection algorithms.*

1 Introduction

In the Internet, each autonomous system (AS) adopts its own local routing policies to choose interdomain routes to achieve objectives such as cost reduction, revenue maximization, latency reduction, and congestion avoidance. The discovery (*e.g.*, [40]) that the interaction of local routing policies (called local policies for short in this paper) can lead to instability has led to extensive research on the subject lately. By instability in this paper, we mean persistent route oscillations even when the network topology is stable. In particular, Griffin *et al.* [17, 21, 22, 25, 36] study the stability of path-vector, policy-based interdomain routing, and identify sufficient conditions to guarantee stability. Gao and Rexford [16, 17] prove that the constraints imposed on local policies by business considerations can lead to stability. Although the preceding stability results are surprisingly pleasant and elegant, practice poses further challenges in analyzing interdomain routing stability.

First, the previous studies focus on a specific interdomain route selection algorithm (*e.g.*, the BGP-based greedy route selection algorithm such as SPVP [22]). As a result, factors such as route dampening, which are present in routing practice, are not easily allowed in previous analysis. Although conceptually such factors might not change the conclusions of previous analysis, an analytical framework is still missing.

Second, the previous studies focus on local policies which rank only the egress routes; that is, they assume that the local ranking of egress routes at each autonomous system is independent of the inbound traffic pattern of the AS. This independence is justified when the inbound traffic of an AS is relatively constant. However, in practice, the local policies of ASes may involve both the egress routes and the pattern of inbound traffic, introducing unexpected interaction.

Specifically, an AS may rank egress routes depending on the pattern of inbound traffic. If this happens, we say that the local policy of the AS depends on the inbound traffic pattern, or inbound traffic for short. We also say that the local policy of the AS is inbound-traffic-dependent, or inbound-dependent for short. Consider a transit AS with two available egress routes, r_1 and r_2, to a major destination. If the total traffic demand to the destination is low, the AS may choose r_1, because it is relatively cheaper; on the other hand, if the traffic demand is high, the AS may choose the more expensive egress route r_2 because it has higher capacity. In other words, the AS essentially has two route ranking tables: one for low traffic volume, in which r_1 is preferred over r_2, and the other for high traffic volume, in which r_2 is preferred over r_1. Since the total traffic demand to the destination depends on inbound traffic for the destination, it introduces dependency between egress routes and inbound traffic. One way such inbound-dependent route se-

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lection can happen is that the operator of the AS observes traffic demand, and manually reconfigures the local preference values of the two routes. Such inbound-dependent route selection can also be implemented automatically, with a traffic engineering algorithm based on an estimated traffic demand matrix. In the last few years, several traffic-demand-matrix-based traffic engineering algorithms have been proposed (*e.g.*, [3, 19]). Although such algorithms have been shown to be effective, the evaluations often assume that the inbound traffic is constant (*e.g.*, the route selection of the AS does not change the inbound traffic). Furthermore, an AS may not only passively react to given inbound traffic, but also actively try to influence the pattern of the inbound traffic (*e.g.*, attracting more customer traffic, and/or load-balancing inbound traffic). The large number of prepended prefixes in BGP routing tables [4] indicates that it is a common practice that ASes try to influence inbound traffic. Recently, we have conducted an email survey of ISPs, and the results indicate that ISPs not only passively react to inbound traffic, but also actively try to influence the pattern of inbound traffic.

In this paper, we analyze the stability of interdomain routing under the general model that the local preference of an AS depends on not only its egress routes to the destinations but also its inbound traffic pattern. Furthermore, instead of studying a specific route selection algorithm, we study a large class of route selection algorithms which are characterized by their asymptotic behaviors.

Specifically, we first show that the common route selection algorithms of choosing the best routes according to the traffic demand matrix of the preceding period could lead to instability, when the route selection of an AS can change its inbound traffic pattern. This instability happens even when all constraints on interdomain routing imposed by business considerations [17] are satisfied, and just a single AS is using such an algorithm. We say that such instability is caused by traffic-route mis-association, and it is an example of instability caused by route selection algorithms. As a remedy, an AS should adopt a route selection algorithm which estimates inbound traffic in such a way that the estimated inbound traffic is truly the result of the chosen egress route.

We then analyze the stability of a network where ASes run any reasonable route selection algorithms which we call *rational route selection algorithms*. The definition of a rational route selection algorithm depends only on the asymptotic behavior of the algorithm. There are several advantages in conducting stability analysis based on the general notion of rational route selection algorithms. First, it allows us to establish stronger positive results in two senses: 1) it allows us to prove the stability of a heterogeneous network where different ASes can run different route selection algorithms, so long all of the algorithms are rational; 2) since the notion of a rational route selection algorithm is defined by its asymptotic behavior, if variations to a route selection algorithm do not change its asymptotic behavior (*e.g.*, non-persistent route dampening), the route selection algorithm is still rational, and thus the stability result still holds. Second, it allows us to establish stronger negative results; for example, if we show that a network is unstable under *any* rational route selection algorithms, it is stronger than to show that a network is unstable under a specific route selection algorithm.

In particular, we derive a sufficient condition to guarantee routing convergence under the general model that the local preference of an AS depends on not only its egress routes to the destinations but also its inbound traffic pattern. This condition applies to any network so long the route selection algorithms of the ASes are rational route selection algorithms. The condition also allows us to predict potential routes. We also show that there exist networks which can have persistent route oscillations even when the local policy of each AS follows the constraints imposed by business considerations, and can adopt *any* one of the rational route selection algorithms. This result clearly demonstrates the intrinsic challenges of route selection for interdomain routing.

The rest of this paper is organized as follows. In Section 2, we discuss related work. In Section 3, we show that the traffic-demand-matrix-based route selection algorithms can lead to routing instability. In Section 4, we define the class of rational route selection algorithms. In Section 5, we present a sufficient condition to guarantee convergence of a network running rational route selection algorithms. In Section 6, we show an example network which is unstable. Our conclusion and future work are in Section 7.

2 Related Work

There is a large body of literature on interdomain route selection. Researchers have conducted extensive evaluations (*e.g.*, [5, 10, 20, 27, 28, 40]) and theoretical analysis (*e.g.*, [17, 21, 22, 24, 25, 36]) on the stability of BGP route selection. In particular, Griffin, Shepherd, and Wilfong [22] show that "policy disputes" can cause persistent route oscillations. Griffin and Wilfong [23] then propose a protocol called SPVP3 that can detect oscillations caused by policy dispute at run time using "path history". SPVP3 is guaranteed to converge if routes whose path history contain cycles are suppressed. Gao and Rexford [16,17] observe that, if every AS considers each of its neighbors as either a customer, a provider, or a peer, and obeys certain local constraints on preference and export policies, then BGP is guaranteed to converge. Generalizing the above commercial relationships of ISPs to a class-based system, Jaggard and Ramachandran [24] show that a global constraint that guarantees convergence can be enforced by a distributed algorithm. Mao *et al.* [32] also describe a mechanism to damp route oscillations. A major difference between our study and the preceding studies is that we study the dependency of route selection on inbound traffic, an important factor which has not been addressed before. Also, we study convergence under general rational route selection algorithms, instead of a specific algorithm.

The interaction of interdomain routing and inbound traffic starts to receive some attention lately [18, 41]. However, the focus of previous studies is on prepending. In [41], Wang *et al.* characterize the stability of inbound-dependent route selection. However, their study focuses on prepending and their specific algorithm. Unlike [41], we focus on route selection, since we feel that the effects of prepending cannot be guaranteed since an AS can choose to ignore the effects of prepending. Also, we investigate the existence and nonexistence of stable route selection for general algorithms, instead of a specific algorithm. To model potential AS behaviors, we adopt a general, rational, learning model. This model is motivated by general game-theoretical, rational algorithms (*e.g.*, adaptive and sophisticated learning algorithms [33]). In particular, our model is inspired by the adaptive learning model of Milgrom and Roberts [33], and the reasonable learning model of Friedman and Shenker [13–15].

Inbound-dependent route selection will be an essential component of general interdomain traffic engineering. However, traffic engineering has traditionally been focused on intra-domain (for a good survey, please see [11, 12]). There is an increasing interest in tuning BGP attributes for interdomain traffic engineering [5, 35]. However, most of the previous work focuses on egress route selection, for either a single AS (*e.g.*, [3,6,19]) or between two neighboring ASes. In particular, researchers have conducted extensive theoretical analysis (*e.g.*, [26]) and experimental evaluations (*e.g.*, [38,39]) of hot-potato routing, which is a scheme of exit route selection between two ASes. Recently, Wang *et al.* study general interdomain egress traffic engineering and identify sufficient conditions for convergence [42]; however, it still focuses only on egress routes only, and adopts a specific route selection algorithm.

Another line of related research is the extensions/alternatives to BGP (*e.g.*, the mechanism-design approach by Feigenbaum *et al.* [7–9], the negotiation protocol by Mahajan *et al.* [29–31], the BGP pricing approach by Afergan and Wroclawski [1], and the Hybrid Link-state Path-vector (HLP) approach of Subramanian *et al.* [37]). The objective of our study is to investigate the intrinsic instability of interdomain routing so that the extensions can guarantee stability under all scenarios.

3 Motivation

3.1 A Motivating Example

We start with an example to show that ASes may adopt general local policies. The example also shows that with inbound-dependency, whether or not a network is stable can depend on the route selection algorithms.

From our email surveys, it is clear that inbound-dependent route selection is important for a transit ISP whose inbound traffic varies substantially with its own route

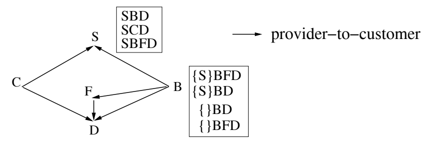

Figure 1. The ranking of egress routes at B depends on inbound traffic. S is the source, and D is the destination.

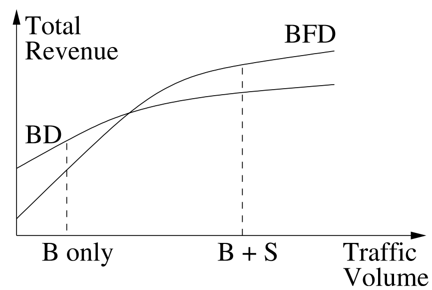

Figure 2. A revenue function justifying the route selection behavior of B in Figure 1. "B only" denotes the traffic volume when S does not use B as its transit provider; and $B+S$ denotes that when S uses B.

selection. Figure 1 shows an example network which is motivated by the increasing usage of multihoming and its potential effects on some transit ISPs. The example network is constructed in such a way that it satisfies all conditions to guarantee stability for inbound-independent route selection [17]: there is no provider-customer loop in the network; each AS follows the *typical export policy*; and an AS prefers customer routes over provider routes. The example network avoids peering links to have a clean setup.

A special feature of this example network, however, is that the ranking of egress routes at B, who is one of the two competing transit providers of source S, depends on its inbound traffic. For generality, we say that B ranks *outcomes*, instead of just egress routes. An outcome consists of both an egress route and ingress traffic pattern. For generality, we assume a ranking table at each AS, which lists, in decreasing order, all of the potential outcomes. Note that in practice, a ranking table can be implemented, compactly, by an objective or utility function. Specifically, $\{S\}BFD$ denotes the outcome that B uses the egress route BFD and S sends traffic for destination D through B; $\{\}BD$ denotes the outcome that B uses the route BD and S does not send any traffic through B.

This example network does not appear to be a pathological case and can well happen in practice. S is a multihomed network with two providers C and B to improve reliability. The ranking table of S is constructed according to the standard BGP decision process: S prefers routes with small AS-hop counts; for two routes with the same AS-hop count, it uses the next-hop ID to break the tie. As for B, when traffic volume is high (*i.e.*, when S uses B as its transit provider), B selects BFD over BD; on the other hand, when traffic volume is low (*i.e.*, when S does not use B as its transit provider), B chooses BD over BFD. A potential revenue function that may cause this scenario to happen is shown in Figure 2; that is, BFD is more profitable for B when the traffic volume is high, while BD is more profitable for B when the traffic volume is low. Note that it is possible to reverse the provider-customer relationship of the AS pairs, CD, FD, BF, and BD. Then the preference of B can be justified by cost instead of revenue.

3.2 Instability of a Traffic-Demand-Matrix-Based Route Selection Scheme

A common approach for B to implement inbound-dependent route selection is to use a traffic-demand-matrix-based algorithm (*e.g.*, [3, 19]). The basic structure of such an algorithm is that time is divided into multiple periods. During each time period, the algorithm measures the traffic demand matrix. At the end of each time period, the algorithm computes and installs the optimal route selection for the next period.

Specifically, B would implement a route selection algorithm as follows. During each time period n, B estimates total traffic demand to destination D; At the end of time period n, B computes the optimal route selection (BFD or BD), based on the measured inbound traffic demand and its traffic engineering objectives. B then installs the optimal route selection at the beginning of time period $n+1$. As we have discussed in the introduction, this algorithm can be implemented either by a network operator manually, which will operate at a longer time scale, or by a traffic engineering program, which will operate at a much faster speed.

Given the above route selection algorithm, assume that B initially chooses egress route BD. B exports BD to S; therefore, S chooses SBD over SCD, and the traffic from S to D goes through B. However, given this high inbound traffic demand, B prefers BFD over BD; thus B switches its route selection to BFD and exports to S. This change of egress route causes S to choose SCD over $SBFD$, and thus traffic of S no longer goes through B. Given that now the inbound traffic is low, B switches back to route selection BD, since it prefers BD over BFD at low traffic. Thus, we have obtained persistent route oscillations[1].

3.3 Optimal and Stable Inbound-dependent Route Selection by a Single AS

The above instability is due to the fact that under the preceding traffic-demand-matrix-based route selection algorithm, B mis-associates the outcomes with its available actions (*e.g.*, B has two available actions in the preceding example: choosing BD or BFD). To choose the optimal route and maintain stability, an AS i needs to correctly associate the outcomes with its actions; that is, the estimated inbound traffic pattern is a result of the chosen egress route. Learning the outcomes of all available egress routes, AS i chooses the optimal outcome. Figure 3 specifies a route selection algorithm which can guarantee stability and optimality, when only AS i adopts this inbound-dependent route selection algorithm. Note that in Figure 3, r_i is a route selection constructed from the routes exported by AS i's neighbors. We refer to that a route profile r_i is overwhelmed by r'_i if (1) whenever r_i is available, r'_i is also available; and (2) choosing r'_i always yields strictly more preferable outcome than choosing r_i. This notion will be formalized in our general model of route selection algorithms.

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▷ T_conv is maximum time for routing convergence
▷ T_m is the measurement time
▷ R_i is the set of available, unoverwhelmed route selections
      constructed from routes exported by i's neighbors
▷ tm(r_i) represents the inbound traffic matrix when choosing r_i

foreach r_i in R_i
        install r_i
        estimate tm(r_i) by
                waiting for T_conv
                measuring tm(r_i) for T_m
        if any route in R_i is overwhelmed
              remove it from R_i
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Figure 3. An inbound-dependent route selection algorithm.

Specifically, in the context of Internet interdomain route selection, when ASes are constrained by Internet business considerations, Theorem 1 shows that the algorithm in Figure 3 can guarantee stability and optimality. Due to space limitation, we omit its proof, and note that an induction proof can be constructed.

Theorem 1 *The network converges, and an AS i converges to its optimal outcome, if the following conditions are satisfied:*

1. *there is no provider-customer loop in the network;*
2. *all ASes except i adopt the typical export policy;*
3. *each AS prefers customer routes over peer/provider routes;*
4. *AS i adopts the route selection algorithm in Figure 3, and no other AS uses any inbound-dependent route selection.*

Consider the example in Figure 1. At the beginning, B does not know which of BD or BFD is the egress route to choose. So it can select either one. Later, it learns the

[1]This oscillation is different from that generated by classical single-path adaptive routing; for example, the classical routing scheme considers only latency [2].

outcomes of choosing BD and BFD. Since the outcome of choosing BD is more preferred than that of choosing BFD, B chooses BD. For brevity, we also say that BD overwhelms BFD.

4 General Rational Route Selection Algorithms

It is clear from the preceding section that the stability of a network depends on not only the interaction of the local routing policies of the ASes in the network, but also the route selection algorithms implementing the policies. The instability studied by the previous studies is caused by policy interaction, while the instability identified in the preceding section is caused by the specific route selection algorithm. Since it is highly likely that more route selection algorithms will be designed, it is important to analyze the stability of a heterogeneous network where ASes run any reasonable route selection algorithms, not a homogeneous network where all ASes run a single, specific algorithm, for example, the greedy BGP algorithm, or the one in Figure 3. This is particularly important in a network when ASes adopt different types of local policies (*e.g.*, some inbound independent while some dependent), since it is reasonable that then different ASes may choose route selection algorithms according to their local policies.

Below, we define the notion of *rational route selection algorithms*. The concept of rational route selection algorithms is motivated by previous work on adaptive learning [33] and learning on the Internet [14]. The models used in the previous game theoretical studies are normal form games. However, interdomain route selection is more of an extensive form game than a normal form game, since an intrinsic characteristic of interdomain route selection is that the available routes of an AS depend on those exported by its neighbors. In this paper, we shall explicitly model this dependency. In the sequel, we shall formalize our intuitive notion of rational route selection algorithms and explore the implications.

4.1 Rational Route Selection: Model

The network topology is represented by a simple, undirected graph $G = (V, E)$, where $V = \{1, \ldots, N\}$ is the set of ASes and E is the set of interdomain links.

A path in G is either the empty path, denoted by ϵ, or a sequence of ASes $(v_k, v_{k-1}, \ldots, v_1, v_0)$, where $k \geq 0$ is the length of the path, such that $(v_i, v_{i-1}) \in E$ for $i = k, k-1, \ldots, 1$. Note that if $k = 0$, then (v_0) represents the trivial path from v_0 to itself. Each nonempty path $P = (v_k, v_{k-1}, \ldots, v_1, v_0)$ has a direction from v_k to v_0. If P and Q are two nonempty paths such that the first AS in Q is the same as the last AS in P, then PQ denotes the path formed by the *concatenation* of these two paths. We extend this with the convention that $\epsilon P = P\epsilon = P$ for any path P.

We denote by R the set of all paths in G. For each $i \in V$, we denote by $R_{i\rightarrow}$ the set of paths originating from i, and by $R_{\rightarrow i}$ the set of paths terminating at i. Also, for any $i, j \in V$, $R_{i\rightarrow j} = R_{i\rightarrow} \cap R_{\rightarrow j}$ denotes the set of paths from i to j.

Suppose i and j are two neighboring ASes. As a path P is exported from j and imported into i, it undergoes two transformations. First, $P_1 = \text{export}(i, j, P)$ represents the application of export policies of j to P, which includes possibly prepending j multiple times to P or filtering out P altogether ($P_1 = \epsilon$). Second, $P_2 = \text{import}(i, j, P_1)$ represents the application of import policies of i to P_1. In particular, import policies at i will filter out any path that contains i itself ($P_2 = \epsilon$). The collective effects of these transformations can be represented by the *peering transformation*, $\text{pt}(i, j, P)$, defined as

$$\text{pt}(i,j,P) = \begin{cases} \text{import}(i,j,\text{export}(i,j,P)) & \text{if } (i,j) \in E, \\ \epsilon & \text{otherwise.} \end{cases}$$

The peering transformation represents the import/export policies of all ASes in the network. Note that in the above definition, we extend the domain of pt to all pairs of ASes by setting $\text{pt}(i, j, P) = \epsilon$ if i and j are not neighbors.

Each AS $i \in V$ has a set $\mathcal{D}_i \subseteq V$ of destinations, and attempts to establish a path to each destination $j \in \mathcal{D}_i$. A *network route selection* is a function r that maps each pair of ASes $i \in V$ and $j \in \mathcal{D}_i$ to a path $r(i, j) \in R_{i\rightarrow j}$. We interpret $r(i, j) = \epsilon$ to mean that i is not assigned a path to j. We denote by $\mathcal{R}$ the set of all possible network route selections. When we restrict our attention to the route selection of AS i alone, we shall refer to the restriction of r on i and $\mathcal{D}_i$ as the *route profile* for AS i, denoted by r_i. We denote by $\mathcal{R}_i$ the set of all possible route profiles for AS i. Note that in the above definition, we do *not* require the routes in a network route selection to be consistent; that is, if $r_i(k) = (i, j)P$, it is not necessary that $r_j(k) = P$.

The above definitions lead to useful equivalent representations of network route selections and route profiles. First, a network route selection r can be represented as $r = (r_i, r_{-i})$, where $r_{-i} = (r_j)_{j\neq i}$ denotes the *combined route profiles* of all ASes except i. The route profile of AS $j \neq i$ in r_{-i} is denoted by $(r_{-i})_j$. We denote by $\mathcal{R}_{-i}$ the set of all possible combined route profiles of all ASes except i; that is, $\mathcal{R}_{-i} = \{r_{-i} | (r_{-i})_j \in \mathcal{R}_j, \forall j \neq i\}$. Second, network route selections and (combined) route profiles can be treated as sets of paths. Specifically, a network route selection r, a route profile r_i and a combined route profile r_{-i} are equivalent to the sets of paths $\{r(i, j) | i \in V, j \in \mathcal{D}_i\}$, $\{r_i(j) | j \in \mathcal{D}_i\}$, and $\{(r_{-i})_j(k) | k \in \mathcal{D}_j, j \neq i\}$, respectively. This equivalent representation is particularly convenient in some operators defined on sets of paths. For example, we can simply use r_{-i} as an argument to such an operator, where actually the argument is $\{(r_{-i})_j(k) | k \in \mathcal{D}_j, j \neq i\}$.

An intrinsic characteristic of path vector protocols such as BGP is that there are dependencies among route selections of ASes. Specifically, the route profiles available to

i depend on the route advertisements it receives from its neighbors, which in turn depend on route selections of these neighbors. To capture this dependency, we define two operators C_i and A_i for each AS i as follows. For a set of paths $\mathcal{P} \subseteq R$, let

$$C_i(\mathcal{P}) = \{(i,j)\,\mathrm{pt}(i,j,P) | P \in \mathcal{P} \cap R_{j\rightarrow}\} \quad (1)$$

$$A_i(\mathcal{P}) = \{r_i \in \mathcal{R}_i | r_i(k) \in C_i(\mathcal{P}) \cup \{\epsilon\}, \forall k \in \mathcal{D}_i\} \quad (2)$$

Intuitively, if $\mathcal{P}$ is the set of routes exported by i's neighbors, then $C_i(\mathcal{P})$ is the set of routes available to i in its routing cache, and $A_i(\mathcal{P})$ is the set of route profiles that i can possibly choose from this routing cache. Note that AS i can always choose the empty path to any $k \in \mathcal{D}_i$ regardless of $C_i(\mathcal{P})$.

The route selection objective of AS i (*i.e.*, its local preference) is represented by a utility function $u_i(r_i, r_{-i})$, which evaluates the payoff of the current network route selection r for i. Note that since we allow the utility of i to depend on not only i's route, but also all other ASes' routes, it captures inbound-dependent route selection.

As is mentioned at the beginning of this section, we want to analyze the stability of a heterogeneous network where ASes run any reasonable route selection algorithms. In order to achieve this generality, we avoid any detailed specification of how the ASes actually select route profiles. Instead, we focus on the sequence of network route selections over time, and identify some general properties fulfilled by these sequences when ASes use any reasonable route selection algorithms that we consider.

We assume that there is a set of times $T = \{0, 1, 2, \ldots\}$ at which one or more ASes in the network change their route profiles. The elements of T should be viewed as the indices of the sequence of physical times at which these changes take place. At time t, the selected route profile of AS i is $r_i[t]$, and the network route selection is $r[t] = (r_i[t])_{i \in V}$. The sequence of network route selections is, therefore, $\{r[t]\}_{t=0}^{\infty}$.

Given a set $H \subseteq \mathcal{R}$ of network route selections, we define the *projection* of H onto $\mathcal{R}_i$ as

$$H_i = \{r_i \in \mathcal{R}_i | r \in H\}. \quad (3)$$

Accordingly, we define the *product* set H_{-i} as

$$H_{-i} = \{r_{-i} \in \mathcal{R}_{-i} | (r_{-i})_j \in H_j, \forall j \neq i\}. \quad (4)$$

The set H_{-i} represents all possible combined route profiles of all ASes except i, where AS j's route profile is drawn from H_j for all $j \neq i$. Also, let

$$A_i(H_{-i}) = \bigcup_{r_{-i} \in H_{-i}} A_i(r_{-i}). \quad (5)$$

Recall that in the above definition, $A_i(r_{-i})$ actually means $A_i(\{(r_{-i})_j(k) | k \in \mathcal{D}_j, j \neq i\})$.

Suppose that AS i has observed a set H of network route selections, and believes that each other AS j will select route profiles in H_j. It is reasonable, therefore, for i to believe that the route selections of the other ASes belong to the set H_{-i}, and that the route profiles possibly available to it will belong to the set $A_i(H_{-i})$. If there exist two route profiles $r_i, r'_i \in A_i(H_{-i})$, such that

1. whenever r_i is available, r'_i is also available;
2. choosing r'_i always yields strictly higher payoff than r_i;

then it would be "unjustified" or "irrational" for i to choose r_i. Formalizing the above argument, we define the following operator $U : 2^{\mathcal{R}} \mapsto 2^{\mathcal{R}}$:

Definition 1 *Given $H \subseteq \mathcal{R}$, let*

$$\begin{aligned} U_i(H) = \{ & r_i \in A_i(H_{-i}) | \forall r'_i \in A_i(H_{-i}), P1 \vee P2, \\ & \textit{where} \\ & \quad (P1)\ \exists r_{-i} \in H_{-i}, \textit{such that} \\ & \qquad r_i \in A_i(r_{-i}), r'_i \notin A_i(r_{-i}), \\ & \quad (P2)\ \exists r_{-i} \in H_{-i}, \textit{such that} \\ & \qquad r_i \in A_i(r_{-i}), r'_i \in A_i(r_{-i}), \\ & \qquad u_i(r_i, r_{-i}) \geq u_i(r'_i, r_{-i})\}, \\ U(H) = \{ & r \in \mathcal{R} | r_i \in U_i(H)\}. \end{aligned}$$

The set $U_i(H)$ is the set of route profiles that are not *overwhelmed* when each other AS j is limited to route profiles in H_j. If AS i believes that other ASes will select route profiles in H_{-i}, then it would be "unjustified" for AS i to choose any route profile not in $U_i(H)$, since every such route profile is guaranteed to be strictly worse than some other route profile in $U_i(H)$. $U_i(H)$ thus formalizes our notion of the set of *unoverwhelmed* route profiles for AS i that are consistent with the route selections of other ASes H_{-i}.

Our intuitive notion of "rational route selection" is defined in terms of properties fulfilled by a sequence of network route selections.

Definition 2 $\{r_i[t] | t \in T\}$ *is* consistent with rational route selection *if, for all t', there exists $t'' > t'$ such that for all $t > t''$, $r_i[t] \in U_i(\{r[s] | t' \leq s < t\})$. $\{r[t] | t \in T\}$ is* consistent with rational route selection *if each $\{r_i[t] | t \in T\}$ has this property.*

4.2 Example: BGP for Inbound-independent Interdomain Routing

The preceding definition of rational route selection is generic and does not specify how ASes actually select route profiles. Thus, it allows both centralized and distributed implementations. An example centralized implementation can be as follows. Each AS sends its utility function to a trusted third party. The third party then applies the operator U to compute for each AS a routing schedule (namely what route each AS should adopt at what time).

As an example of distributed implementation, below we analyze the *standard BGP route selection protocol* as it is used in interdomain route selection. By the standard BGP route selection protocol, we mean essentially the simple path vector protocol (SPVP) as defined in Fig. 5 of [22], extended to the case of joint multiple-destination route selection, and other features such as route dampening, so long some mild conditions are satisfied. We will show that the asymptotic best-response nature of BGP makes it a rational route selection algorithm, when the ranking of egress routes is independent of inbound traffic.

Specifically, we have the following result:

Theorem 2 *The BGP protocol is consistent with rational route selection, if the following conditions are satisfied:*

A1. BGP update messages between neighboring ASes are delivered reliably in FIFO order, and have bounded delay;

A2. Each AS sends out BGP update messages in bounded time after it updates its route profile;

A3. Each BGP update message is processed immediately.

Proof: Let the sequence of network route selections be $\{r[t]\}_{t=0}^{\infty}$.

Consider an arbitrary AS i. Let $\mathcal{N}_i$ be the set of neighbors of i. For any $j \in \mathcal{N}_i$, let $r_j[\tau_j^i(t)]$ be the latest route profile of j such that an update message has been sent to i with this route profile. Thus $C_i(r_j[\tau_j^i(t)])$ is the set of paths in i's routing cache learned from j at time t. The set of route profiles available to i is therefore $A_i(\{r_j[\tau_j^i(t)] | j \in \mathcal{N}_i\})$. Assumptions A1 and A2 imply that there exists t_d such that at any time t, for any neighbor j of i, $\tau_j^i(t) \geq t - t_d$.

Although i may not know $r_{-i}[t]$, the payoff $u_i(r_i, r_{-i})$ is only a function of r_i. (Recall that we consider only egress route selection in this case.) The BGP protocol, together with Assumption A3, implies that at any time t

$$r_i[t] = \arg\max_{r_i \in A_i(\{r_j[\tau_j^i(t)] | j \in \mathcal{N}_i\})} u_i(r_i, r_{-i}[t]). \qquad (6)$$

We shall prove the theorem by showing that $t'' = t' + t_d$ satisfies Definition 2. In fact, for any $t > t''$, let $H = \{r[s] | t' \leq s < t\}$. For any neighbor j of i, we have $\tau_j^i(t) \geq t - t_d \geq t'$, thus $r_j[\tau_j^i(t)] \in H_j$. Therefore, there exists $r_{-i} \in H_{-i}$ such that $r_j[\tau_j^i(t)] = (r_{-i})_j$. We shall show that $r_i[t] \in U_i(H)$. We have that $r_i[t] \in A_i(r_{-i}) \subseteq A_i(H_{-i})$. For any $r_i' \in A_i(H_{-i})$, if predicate P1 does not hold, then $r_i' \in A_i(r_{-i})$, which, together with Equation (6), implies that $u_i(r_i[t], r_{-i}[t]) \geq u_i(r_i', r_{-i}[t])$. It follows that $r_i[t] \in U_i(H)$. ∎

Remark 1 *These three assumptions of the theorem should be valid under normal network operations. For example, when an AS applies route dampening, if the amount of time that a route is dampened has a finite upper bound, then the assumptions are still valid.*

Remark 2 *Note that in Definition 2, AS i is not required to know the route selections $r_{-i}[t]$ of the other ASes. AS i may not even know the sequence of times T and its set of all possible route profiles $\mathcal{R}_i$. In addition, the definition says nothing about the routing cache of i. The $r_{-i} \in H_{-i}$ used in Definition 1 may have never appeared in i's routing cache from time t' up to t. Moreover, at some time t, $r[t]$ may not even be consistent. All that is required is that the exhibited sequences of route selections $r_i[t]$ and $r[t]$ satisfy the requirement in the definition. The preceding theorem is an example clarifying this subtlety.*

5 A Sufficient Condition to Guarantee Convergence of Rational Route Selection Algorithms

Given the definition of rational route selection algorithms, in this section, we derive a sufficient condition to guarantee stability. The advantage of deriving a sufficient condition using the general notion of rational route selection algorithms is that we then only need to consider the asymptotic behaviors of route selection algorithms, allowing variations such as route dampening and limited route experimentation.

We first define the notion of stable route selection.

Definition 3 *A network consisting of ASes each of which is running a rational route selection algorithm has a* stable route selection*, if the route selection of each AS has a single route profile, as time goes to infinite. Formally, the network has a stable route selection if $\{r[t]\}_{t=0}^{\infty}$ converges.*

Remark 3 *In the above definition, we require that, in a stable route selection, the route selection of each AS be a "pure" routing decision. We do not allow "mixed" strategies [34], since mixed strategies involve frequent route fluctuations, and are thus not desirable as "stable" solutions for global interdomain routing.*

We first observe the following important property of the operator U:

Lemma 3 *The operator U is monotone: If $P, Q \subseteq \mathcal{R}$ and $P \subseteq Q$, then $U(P) \subseteq U(Q)$.*

Proof: It suffices to show that $U_i(P) \subseteq U_i(Q)$ for an arbitrary i.

Suppose $r_i \in U_i(P)$. We first notice that, since the operator A_i as defined in (2) is monotone, $r_i \in A_i(P_{-i})$ implies $r_i \in A_i(Q_{-i})$. To prove $r_i \in U_i(Q)$, we only need to show that, for any $r_i' \in A_i(Q_{-i})$, at least one of the two predicates P1 and P2, which are defined in Definition 1, holds. We distinguish the following two cases:

1. $r_i' \in A_i(P_{-i})$. In this case, the fact that $r_i \in U_i(P)$ implies that at least one of the two predicates P1 and P2 holds.

2. $r'_i \notin A_i(P_{-i})$. This case happens only if $\forall r_{-i} \in P_{-i}, r'_i \notin A_i(r_{-i})$. Thus predicate P1 holds in this case.

∎

We now observe that sequences consistent with rational route selection share some common asymptotic properties:

Theorem 4 *If $\{r[t]|t \in T\}$ is consistent with rational route selection, then for each k, there exists $t_k \in T$ such that, for all $t \in T$ with $t \geq t_k$, $r[t] \in U^{(k)}(\mathcal{R})$.*

Proof: For $k = 0$, the conclusion holds trivially (choosing $t_0 = 0$) since for all t, $r[t] \in \mathcal{R} = U^{(0)}(\mathcal{R})$.

Suppose the conclusion holds for $k-1$. Then, there is a t_{k-1} such that for all $t \geq t_{k-1}$, $\{r[s]|t_{k-1} \leq s \leq t\} \subseteq U^{(k-1)}(\mathcal{R})$. Since $\{r[t]|t \in T\}$ is consistent with rational route selection, in Definition 2 we may choose $t' = t_{k-1}$ and we may take $t_k > \max(t'', t_{k-1})$. Therefore, for all $t \geq t_k$, we have that $r[t] \in U(\{r[s]|t_{k-1} \leq s < t\}) \subseteq U(U^{(k-1)}(\mathcal{R})) = U^{(k)}(\mathcal{R})$. ∎

By Theorem 4, when the serially unoverwhelmed set $U^{\infty}(\mathcal{R})$ is small, one can predict with precision the asymptotic behavior of a sequence of network route selections. In particular, if $U^{\infty}(\mathcal{R})$ is a singleton, Theorem 4 immediately implies that the sequence will always converge to a unique network route selection. We therefore extend similar results in the context of strategic learning game [33] and learning in the Internet [14] to our route selection context.

Proposition 5 *The network route selection of a network consisting of ASes running rational route selection algorithms asymptotically lie in the set $U^{\infty}(\mathcal{R})$. Thus, if $U^{\infty}(\mathcal{R})$ is a singleton, the network is guaranteed the existence and uniqueness of stable route selection.*

One way to guarantee that $U^{\infty}(\mathcal{R})$ is a singleton is the existence of a sequentially dominant route selection (SDRS). By a sequentially dominant route selection, we mean a partial order of the ASes, with the destination being the first one, such that given the route selection of the ASes before i in this partial order, the best route selection of i is determined, independent of the route selection of those after i. If a network has an SDRS, all routes other than the unique solution are not in the unoverwhelmed set. As such, $U^{\infty}(\mathcal{R})$ is a singleton. The convergence of such networks under any rational route selection algorithms, therefore, follows immediately from Theorem 2 and Proposition 5. Note that the existence of SDRS can be checked in polynomial time.

As an application of the preceding results, we derive a sufficient condition to guarantee routing convergence in a heterogeneous network where each AS runs any rational route selection algorithm, and its egress route selection satisfies the constraints imposed by business considerations [17].

Theorem 6 *Assume a network where each AS runs any rational route selection algorithm, and selects egress routes independent of inbound traffic. Assume that 1) there is no provider-customer loop in the network; and 2) each AS adopts the typical export policy and the standard joint-route preference [42]. Then $U^{\infty}(\mathcal{R})$ is a singleton; that is, the network is guaranteed to converge to the unique stable route.*

Proof: *(sketch)* When the conditions of the theorem are satisfied, we can use an induction proof to show the existence of an SDRS. Therefore, the network is guaranteed to converge to the unique stable route. ∎

Remark 4 *The preceding convergence result is more general than that proved in previous studies in that it is not limited to just homogeneous networks where each AS has to run the greedy, best-response BGP algorithm. Other actions, such as non-persistent experimentation, non-persistent dampening, are allowed.*

6 Instability of Networks under any Rational Route Selection Algorithms

Unfortunately, with inbound-dependency, there exist networks which have no stable route selection under any rational route selection algorithms; that is, we can arbitrarily assign route selection algorithm to each AS, so long each algorithm is a rational route selection algorithm, the network has no stable route selection.

In particular, Figure 4 is such an example network. Similar to the network in Figure 1, this network is constructed to satisfy all constraints imposed by AS business considerations; thus, if there were no inbound dependency, the network has a unique stable route selection [17]. Also similar to the network in Figure 1, this network does not appear to be a pathological case and can well happen in practice. Note that this network is a heterogeneous network, where the ranking of routes at S is inbound independent; while A and B are inbound dependent.

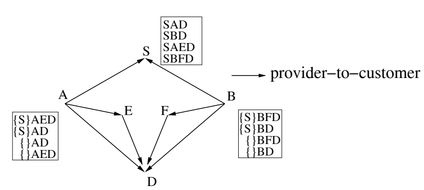

Figure 4. An example with instability. D is the only destination.

The instability of the example network in Figure 4 under any rational route selection scheme is established by the following result:

Theorem 7 *Suppose that a sequence of network route selections $\{r[t]\}_{t=0}^{\infty}$ is consistent with rational route selection* *and that it converges to a stable route selection r^*. Then the following holds for each AS i:*

$$\forall r'_i \in A_i(r^*_{-i}), u_i(r^*_i, r^*_{-i}) \geq u_i(r'_i, r^*_{-i}).$$

Proof: Since $\{r[t]\}_{t=0}^{\infty}$ converges to r^*, there exists t' such that $\forall t \geq t', r[t] = r^*$. Since the sequence is consistent with rational route selection, there exists $t'' > t'$, such that $\forall t > t''$ and $\forall i, r_i[t] \in U_i(\{r[s] | t' \leq s < t\})$. Notice that $\{r[s] | t' \leq s < t\} = \{r^*\}$, by definition of U_i, we have that

$$\forall r'_i \in A_i(r^*_{-i}), u_i(r^*_i, r^*_{-i}) \geq u_i(r'_i, r^*_{-i}).$$

■

An analysis of all of the possible network route selections of the example in Figure 4 shows that no network route selection satisfies the condition in Theorem 7. As a result, the network cannot converge to a stable route selection, under any rational route selection algorithm.

To further understand the example, consider the dynamics. When A and B choose AD and BFD. The outcome is SAD since S ranks SAD higher than $SBFD$. Then A has incentive to change from AD to AED since A ranks $\{S\}AED$ higher than $\{S\}AD$. However, B realizes that, it can achieve a better outcome by changing BFD to BD since S will choose SBD over $SAED$. This in turn triggers A to switch from AED back to AD. Thus we end up with A chooses AD and B chooses BFD again, and the process continues forever.

7 Conclusions and Future Work

In this paper, we have conducted the first systematic analysis on the stability of interdomain route selection where an AS's ranking on routes depends on inbound traffic. We have shown that the common scheme of choosing the best routes according to the traffic-demand matrix of the preceding period could lead to instability, when the inbound traffic depends on route selection. We have proposed the notion of rational route selection algorithms, where inferior routes are iteratively eliminated. We derive a sufficient condition to check the stability of a network. We have also shown that there exist networks where routing will be unstable under any rational route selection algorithms, even when the ASes strictly follow the constrains imposed by AS business considerations.

The unstable network shown in Section 6 is particularly troubling in that it does not appear to be a pathological case, and thus could happen in practice. When we encounter such an unstable network setting in practice, there is still no satisfactory solution. Fundamentally, to stabilize the network, tradeoff between local optimality and global stability must be made. Thus to design a stable route selection protocol, the ASes in a network must be willing to look into the future, form the right coalition, and sacrifice short-term benefits. Previous work such as route suppression (*e.g.*, [23]) and route dampening (*e.g.*, [32]) represents interesting potential directions. However, how to design interdomain routing protocols where the tradeoff between stability and local optimality is explicitly made in an incentive-compatible way is still a major remaining challenge.

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